Journal of
Mechanical
Science and
Technology

Journal of Mechanical Science and Technology 21 (2007) 2011~2022

An identification method for joint structural parameters using an FRF-based substructuring method and an optimization technique

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(Manuscript Received February 20, 2007; Revised June 26, 2007; Accepted June 27, 2007)

Abstract

A new method is proposed to identify the joint structural parameters of complex systems using a frequency response function (FRF)-based substructuring method and an optimization technique. The FRF method is used to estimate the joint parameters indirectly by minimizing the difference between the reference and calculated responses using a gradient-based optimization technique with analytical gradient information. To assess the robustness of the identification method with respect to noisy input data, FRFs contaminated by uniformly distributed random noise were tested in a numerical example. The effects of the random noise and the magnitude of the connection stiffness values on the accuracy of the method were investigated while identifying the joint parameters. When the FRFs were contaminated with random noise, the proposed procedure performed well when used to identify the stiffness values, but the accuracy of identification is deteriorative when used to identify the damping coefficients. The joint parameters of a real bolted structure were also identified by the proposed method. The results show that it can be applied successfully to real structures, and that a hybrid approach using both calculated and measured FRFs in the substructure model can enhance the quality of the identification results.

Keywords: Joint parameter identification, Sensitivity analysis, FRF-based substructuring method, Bolted structure

1. Introduction

In structural vibration problems, numerical techniques such as finite element (FE) analysis have become common tools owing to recent advances in methods and the ease of access to commercial software. However, for complex structures, it is still very difficult to predict the responses of dynamic systems using numerical methods because of uncertainties that arise from the material properties, geometry, applied loads, and boundary conditions, including joint characteristics. Among these, the characteristics of mechanical joints have large effects on system responses. However, it is generally difficult to determine the exact dynamic characteristics of joints. In particular, complex systems consist of many subsystems and various mechanical joints such as bolts, rivets, and bushes that are interconnected. For these cases, engineers make equivalent models of the joints that consist of springs, masses, and dampers. Therefore, the identification of joint parameters such as the stiffness of the springs and the damping coefficients of the dampers is one of the most important components required to develop a useful model of complex systems by using numerical methods.

Many researchers have proposed identification methods for joint parameters. The first category of these studies involves updating the FE models directly to reproduce experimental modal analysis results [1-6]. These methods combine FE models with experimental models to estimate the actual joint parameters, or the mass and stiffness matrices. The mass and stiffness matrices are necessary to create updating

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schemes. However, these schemes are very expensive to obtain for complex systems. Moreover, because the experimental models are deduced from a curve-fitting process, one cannot avoid approximation errors.

Meanwhile, other researchers have developed a second category of joint identification methods that are based on experimental data. These can be classified as either modal-based methods [4-6] or frequency response function (FRF)-based methods [7-18]. Modal-based methods use modal parameters such as eigenvectors and natural frequencies to estimate the joint parameters, while FRF-based methods estimate the joint parameters directly from frequency response functions. Recently, the FRF-based method has gained in popularity because the experimental modal model is expensive to use and has relatively low accuracy compared to the FRFs themselves. In FRF-based methods, the joints are modeled with stiffness elements and dashpots, and the parameters are identified from the FRFs of the overall system and substructures by letting the parameters agree with the measured system responses. Tsai and Chou [7] proposed an identification formulation that was based on the receptance method to calculate the properties of a single bolt joint directly from the measured FRFs of structures. Wang and Liou [8, 9] and Ren and Beards [12] suggested joint parameter identification methods that can avoid noise problems from contaminated FRFs by using the FRFs of whole structures and substructures or linear transformation matrices and weighting functions. Ratcliffe and Lieven [13] generalized Ren and Beards's formulation. Ren and Beards [15] also treated a joint identification problem in which the joint parameters' values had large differences in magnitude. Wang and Chuang [18] extended their work to the non-Gaussian noise problem and joints with parameter values that had differences measured in orders of magnitude. Hong and Lee [10] developed a joint parameter identification method that combines the use of measured incomplete FRFs and FRFs computed from a finite element model. Yang and Park [11] proposed an iterative method to identify the joint parameters of a structure from a subset of the FRFs of substructures. Hwang [14] derived a joint parameter expression by comparing FRFs with and without connections. Also, Yang *et al.* [17] identified both translational and rotational stiffness values using a substructure synthesis method and frequency response functions.

The FRF-based methods described above are direct methods that calculate the joint parameters immediately from the equations obtained by manipulating the FRF matrices. The matrix expressions require some inverse operations. As a result, FRF-based direct methods are inherently sensitive to noise contained in the FRFs. Therefore, an indirect method may improve the results. Furthermore, the author and his colleague [19, 20] have developed a design sensitivity formulation using the frame of an FRF-based method and applied it to an engine mount design optimization problem. Reversing the viewpoint of the problem formulation, in this paper we propose a new method that utilizes FRFs and an optimization technique to identify joint parameters. This will maintain the advantages of the FRF-based method but use an indirect method to overcome the noise problems. The proposed method is robust to random noise because noise effects are reduced through the indirect formulation. In addition, the proposed formulation uses a multi-domain FRF-based substructuring (FBS) technique that does not limit the number of connections and substructures to be identified, whereas many methods are confined to two substructures. The FRF-based substructuring method is based on the system response obtained by using the FRFs of the substructures. Therefore, experimental and calculated FRFs can be easily combined. Moreover, the introduced optimization technique enhances our flexibility in identifying the joint parameters. For example, both the average and best approximated values of a parameter for several joints can easily be estimated, or frequency-dependent parameters can be estimated by adding constraints to the optimization problem.

In this paper, joint structural parameters of a real bolted structure are identified by using an FRF-based substructuring method and a gradient-based optimization technique. Before the identification of real-structure parameters, numerical experiments are used to explore the accuracy of the identification procedure with respect to random noise contained in the FRFs and the relative magnitude of the joint stiffness values. Finally, the identification method is applied to a real structure.

2. Identification of joint parameters

As stated in reference [12], the basic idea adopted in almost every FRF-based joint identification method is similar: the joint parameters are determined by minimizing the difference between the measured reference response from tests and the calculated response, which is a function of the joint parameters. The problem is how to find a solution that makes the difference zero. Many researchers extract the joint parameter expressions to be identified using a least-square error minimization approach. However, because the FRFs inevitably contain noise in real situations, one cannot generally find such an exact solution. The best solution must be used instead. Here, we adopt an iterative optimization technique to find the best solution for the joint parameters. The FRF-based substructuring method gives the system response that is to be calculated and compared with the reference response. To enhance the efficiency of the iterations during optimization, an analytic sensitivity formula is proposed and used in the identification procedure.

2.1 FRF-based substructuring method

The structural dynamic system shown in Fig. 1 has n substructures that are connected to each other by mechanical joints. A spring and damper represent each joint, and mass effects are neglected in this study. The reference response is measured at point r, which is located in an arbitrary k-th substructure. The external forces $f^i(\mathrm{i}=1,\cdots,\mathrm{n})$ acting on the substructures are assumed to be known.

The responses at the connecting points must be determined to calculate the reference response by using the FRF-based substructuring method. Assuming that the kinetic energy is transmitted only through the connections between the substructures, the response of the i-th connecting points, x_i^k, on the k-th substructure can be written using the superposition principle as

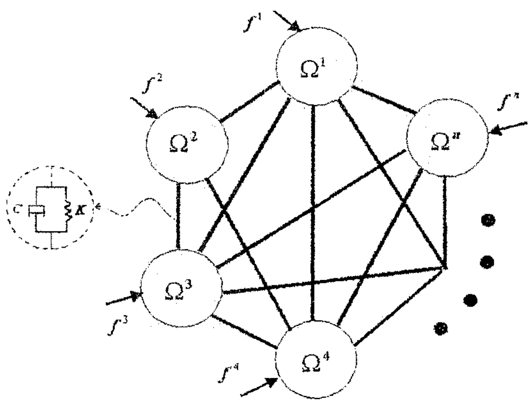


Fig. 1. A substructural system.

$$x_\mathrm{i}^\mathrm{k} = \sum_{j=1, j\neq k}^{n} H_\mathrm{ij}^\mathrm{k} \cdot R_\mathrm{j}^\mathrm{k} + H_\mathrm{if}^\mathrm{k} \cdot f^\mathrm{k}, \quad \mathrm{i}=1,\cdots,\mathrm{n}, \quad \mathrm{i}\neq \mathrm{k} \quad (1)$$

where H_ij^k is the frequency response function matrix between the interfacial points i and j on the k-th substructure, R_j^k is a reaction force vector at the connections acting on the k-th substructure owing to the j-th substructure, and H_if^k is the frequency response function of the i-th interfacial point of the k-th substructure when the external force, f^k, is replaced by a unit force. The reaction forces between two substructures satisfy the force equilibrium equation by Newton's third law as follows:

$$R_\mathrm{i}^\mathrm{k} + R_\mathrm{k}^\mathrm{i} = 0, \quad \mathrm{i,k}=1,\cdots,\mathrm{n}, \quad \mathrm{i}\neq\mathrm{k} \quad (2)$$

If a spring and a viscous damper can be used to represent the characteristics of a connection between substructures, the compatibility equations at the interfacial boundary must be satisfied as follows:

$$S_\mathrm{ki} \cdot R_\mathrm{i}^\mathrm{k} = x_\mathrm{i}^\mathrm{k} - x_\mathrm{k}^\mathrm{i}, \quad \mathrm{i,k}=1,\cdots,\mathrm{n}, \quad \mathrm{i}\neq\mathrm{k} \quad (3)$$

where S_ki is a matrix expression of the compliance coefficients at the interfacial boundary,

$$\begin{aligned}\left(S_\mathrm{ki}\right)_\mathrm{lm} &= \frac{1}{(\mathrm{K_{ki}} + \sqrt{-1}\omega \mathrm{C_{ki}})}, \quad &\text{if} \quad \mathrm{l}=\mathrm{m} \\ &= 0, &\text{if} \quad \mathrm{l}\neq\mathrm{m}\end{aligned} \quad (4)$$

Here, ω is the angular velocity, and $\mathrm{K_{ki}}$ and $\mathrm{C_{ki}}$ are the stiffness and the damping coefficients, respectively, of the connecting elements between the k-th and i-th substructures. Only the diagonal terms of S_ki are not zero, and $\mathrm{K_{ki}}=\mathrm{K_{ik}}$, $\mathrm{C_{ki}}=\mathrm{C_{ik}}$. It should be noted that the stiffness and damping coefficients are not known in the joint parameter identification problem. Thus, an initial assumption for the values is necessary to form the matrix S_ki given in Eq. (4).

Substituting Eqs. (1) and (2) into Eq. (3) and assembling the resulting matrix with respect to the unknown reaction forces, we obtain a new algebraic equation as follows:

$$H \cdot R = F \quad (5)$$

where H is a full matrix in which the FRFs between interfacial points and the joint parameters are assembled according to the connection relations of the substructures, $\boldsymbol{F}$ is a known vector related to the external forces, and

$$R=\left\{R_2^1 \quad R_3^1 \quad \cdots \quad R_n^1 \quad R_3^2 \quad \cdots \quad R_{n-1}^{n-2} \quad R_n^{n-2} \quad R_n^{n-1}\right\}^T \tag{6}$$

Thus, one can obtain all reaction forces at the connections by multiplying both sides of Eq. (5) by the inverse of H.

For the response side, since all reaction forces at the joints are known from Eq. (5), the superposition principle gives the target response expression at point r in the k-th substructure, x_r^k, as follows:

$$x_r^k = \sum_{i=1, i\neq k}^{n} H_{ri}^k \cdot R_i^k + H_{rf}^k \cdot f^k \tag{7}$$

where H_{ri}^k and H_{rf}^k are the frequency response functions of the response point when a unit force is exerted on the interfacial boundary points of the k-th substructure and when the external force, f^k, is replaced by a unit force, respectively. This is a brief summary of the FBS method. One could, though, obtain the FRFs through experiments such as impact tests, or through numerical calculations using FE models for each substructure. Three classes of FRFs are necessary to apply the FRF-based substructuring method to complex structures: those from excitation to connection points in the external-force acting substructures (H_{if}^k), those between connection points in each substructure (H_{ij}^k), and those between connection and response points in the response-point-placed substructures (H_{ri}^k). The FRFs between connection points, H_{ij}^k, are assembled according to the connection layout of the structure in the system matrix of Eq. (5), and the assembled matrices are inversed to obtain the reaction forces at the connection points. Usually, the singular value decomposition method is used to minimize the error effects when a matrix is inversed.

2.2 The joint parameter identification procedure

To identify the joint parameters, the idea of this work is that if we have the correct parameter values, the response computed by the FBS method will exactly coincide with the measured overall system level reference response. Thus, one can identify the correct parameters by minimizing the response differences between the reference and computed values using a numerical searching algorithm. To identify the joint parameters of a dynamic structure with a mathematical programming scheme, we require an identification index that is an objective function that becomes zero for the correct parameters and has a form such as

$$\Psi = \Psi\{x_{ref} - x_r^k(b)\} \tag{8}$$

where x_{ref} is a known response of a dynamic structural system and x_r^k is a response calculated at the same point by the FBS method with joint parameters b. Because all substructures remain unchanged during the joint parameter identification procedure, the response in Eq. (8) is only a function of the joint parameters. Therefore, by minimizing the identification index of Eq. (8) within a design space of b, one can estimate the joint parameters of a dynamic system.

Many mathematical programming algorithms are available to minimize the identification index. Among them, the gradient-based algorithm is the most efficient from an engineering viewpoint, even though it may yield a local minimum. The first gradient of the identification index with respect to the parameters is required to use the algorithm. This can be written from Eq. (8) as

$$\frac{d\Psi}{db} = -\frac{\partial \Psi}{\partial x_r^k} \cdot \frac{\partial x_r^k}{\partial b} \tag{9}$$

The partial derivative of x_r^k, $\partial x_r^k / \partial b$, is an implicit function of the structural parameters. To obtain the partial derivative efficiently, we have developed a sensitivity analysis method using the framework of the FBS method and demonstrated its usefulness [20]. The sensitivity analysis formulation in reference [20] is briefly explained here for completeness. The method utilizes a direct differentiation method to calculate the unknown term, $\partial x_r^k / \partial b$. Assuming that the derivative exists, the differentiation of Eq. (7) results in the following equation:

$$\frac{\partial x_r^k}{\partial b} = \sum_{i=1, i\neq k}^{n} \left\{ H_{ri}^k \cdot \frac{\partial R_i^k}{\partial b} \right\} \tag{10}$$

Here, it should be noted that the external forces and the FRFs of the substructures are not changed because only the joint parameters vary during the identification procedure. To obtain the partial derivative, $\partial R_i^k / \partial b$, in Eq. (10) explicitly, Eq. (5) is differentiated with respect to the parameters:

$$\frac{\partial H}{\partial b} \cdot R + H \cdot \frac{\partial R}{\partial b} = 0 \tag{11}$$

Multiplying both sides of Eq. (11) by the inverse matrix of H and rearranging the terms, we obtain

$$\frac{\partial R}{\partial b} = -H^{-1} \cdot \left[\frac{\partial H}{\partial b} \cdot R \right] \tag{12}$$

where

$$\frac{\partial R}{\partial b} = \left\{ \frac{\partial R_2^1}{\partial b} \; \frac{\partial R_3^1}{\partial b} \cdots \frac{\partial R_n^1}{\partial b} \; \frac{\partial R_3^2}{\partial b} \cdots \frac{\partial R_{n-1}^{n-2}}{\partial b} \; \frac{\partial R_n^{n-2}}{\partial b} \; \frac{\partial R_n^{n-1}}{\partial b} \right\}^T \tag{13}$$

The partial derivative of the system matrix H in Eq. (12) can be obtained analytically from Eq. (4). It is noted that the derivative of the non-diagonal elements become zero. Therefore, one can compute the gradient information from Eqs. (9), (10), and (12). The most expensive calculation required to obtain the gradient information is the computation of the inverse matrix, H^{-1}. However, the inverse matrix is already known because the FRF-based substructuring framework is used for the response calculations.

By obtaining the gradient information of the identification index function, one can estimate the structural joint parameters numerically, starting from an initial parameter set, by plugging the gradient information into a gradient-based optimization algorithm. The joint parameters must be initialized from arbitrary values. Then the optimization algorithm yields a solution based on the mathematical programming method using the gradient information. However, this solution may be a local minimum. One must judge whether the solution is acceptable by comparing the reference response with the calculated response and considering the locations of the peaks and the levels of the frequency response functions. If the solution is not acceptable, different initialization values must be selected and the procedure must be repeated until acceptable joint parameters are obtained. One can overcome the local solution problem by introducing an interactive identification procedure, and generally several starting points are sufficient to identify the structural parameters [21]. This is an iterative procedure, which is summarized in Fig. 2.

Many types of functions can be used for the identification index. When selecting the identification index function, one should consider that gradient-based minimization algorithms give local rather than global minimum points. Therefore, for the identification index, it is important to select a monotonically decreasing function that is as wide as possible over the

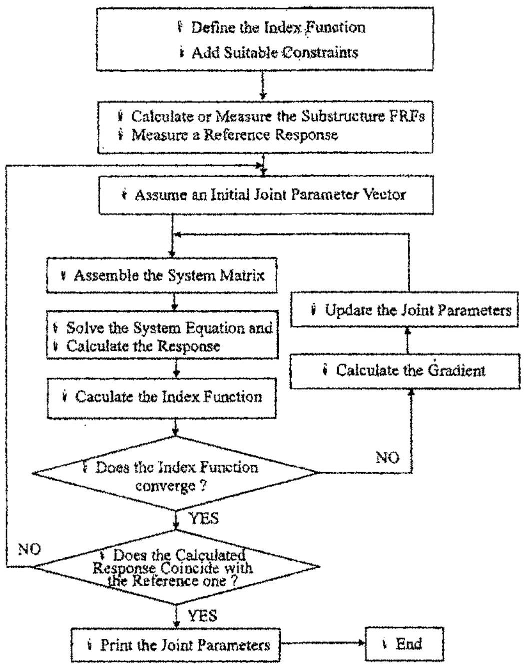


Fig. 2. The joint parameter identification procedure.

parameter range. The authors have explored and tested different types of identification index functions in reference [21]. From a numerical study, we concluded that the square of the magnitude difference between the reference and computed FRFs on a decibel scale gave the most stable identification results.

The proposed method is not restricted to simple two-substructure systems, because a multi-substructure domain is assumed in the formulation. Furthermore, one can easily expand the formulation to frequency-dependent joint parameter problems. The formulation requires only one system level reference response. These advantages enhance the applicability of the proposed formulation.

3. Numerical example and discussion

To demonstrate the usefulness of the proposed joint parameter identification method, the authors explored the accuracy of the method using the numerical example illustrated in Fig. 3. The ladder structure shown in the figure has four rectangular steel beams that are connected to each other by elastic springs and viscous dampers. The goal of this problem was to identify the spring stiffness values and damping coefficients.

In order to identify the structural parameters, an identification index was defined as

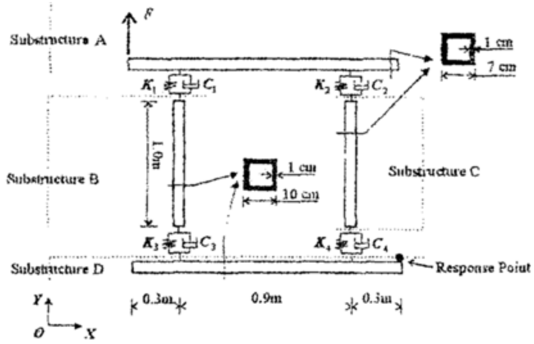


Fig. 3. A ladder structure.

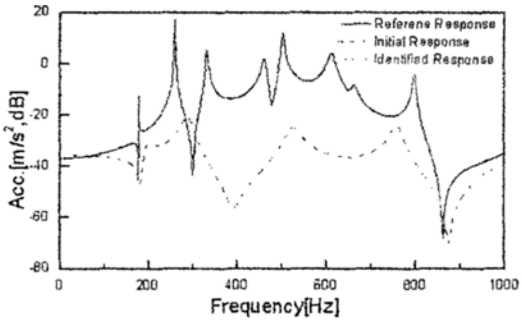
$$\Psi = \int_{f1}^{f2} 400 \ \left(\log\|x_i\| - \log\|x_{ref}\|\right)^2 \mathrm{df} \tag{14}$$

where f and $\| \ \|$ are the frequency and the magnitude of the vector, respectively. The identification index was the squared area formed between the reference and calculated responses along frequency axis. The index function contained an integration process that smoothed the integrand to stabilize noisy responses. This is a very desirable characteristic because the responses could be contaminated by unavoidable noise in experiments. Furthermore, the square operation of the integrand acts as a weighting factor along the frequency axis.

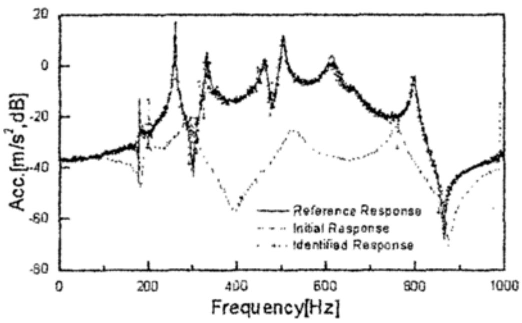
For the FBS method response calculations, the ladder structure was decomposed into four substructures and FE models for the substructures were developed by using ten and fourteen beam elements for the vertical and horizontal substructures, respectively. The frequency response functions of each substructure required in Eq. (7) were calculated from the FE models using MSC/NASTRAN [22]. In this study, the FRFs calculated from FE models were regarded as exact and uncontaminated. An FE model of the entire ladder structure with specified joint parameters was used to calculate the frequency response at the response point shown in Fig. 3; this computed response was used as the reference response for the problem.

3.1 Effects of random noise in the FRFs

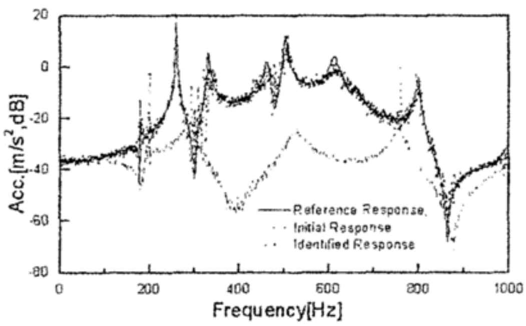
The identification procedure described in Section 2 requires correct FRFs to calculate the correct response of a dynamic system. However, measured FRFs contain noise from various sources, which is inevitable in experiments. Experimental noise contained in FRFs consists of biased noise and unbiased one. Unbiased random noise comes from uncontrollable system



(a) Not contaminated case



(b) 10% noise



(c) 20% noise

Fig. 4. Initial and identified responses (Case IV).

variation. Biased noises are deeply related with improper experimental set-up and techniques, which are beyond our concern in this paper. Thus, the influence of random noise on the identification accuracy of the proposed method was investigated here. To simulate the unbiased noise effects, uniformly distributed random noise with a zero mean value was added to the exact FRFs and the identification procedure was performed by using the contaminated FRFs. The noise added to each FRF can be represented as follows:

$$\bar{h} = h(1+\varepsilon) \tag{15}$$

where h is an FRF and ε is the random noise, which was uniformly distributed in $[-\alpha,+\alpha]$. The magnitude of interval α indicates the maximum noise level contained in the FRFs. The random noise was generated by the **rand** function found in MATLAB software [23].

The accuracy of the identification procedure was tested for the following four cases.

Case *I*: FRFs in H (all H_{ij}^{k}) were contaminated.

Case *II*: FRFs between the external force and connection points (H_{if}^{k}) were contaminated.

Case *III*: FRFs between the connection and response points (H_{ri}^{k}) were contaminated.

Case *IV*: All FRFs were contaminated.

Noise levels of 0, 10, and 20% were imposed on the FRFs of each substructure. Tables 1 – 4 show the identification results. For each case, the correct stiffness and damping coefficient values were 1.0×10^{8} N/m and 1.0×10^{3} N·s/m, respectively. As all cases produced similar results, Fig. 4 shows the initial and identified responses for case *IV* only. The identification procedure reconstructed the reference response well despite the relatively large noise levels. The stiffness values were in reasonable agreement with the expected values, considering the noise levels. However, the identification results for the damping coefficients contained large errors relative to the noise level. This was because the magnitude of the damping has a large influence on the magnitude of the response at the resonance point for cases with low damping. Accordingly, a small contamination of the response near the resonance point results in large errors in the identification of the damping coefficients. However, the random noise imposed in the numerical study was a very severe case, especially near the resonance points. In real situations, the noise level around the peaks is lower in general. By examining the identification results of Tables 1 – 4, one can also see that as the random noise in the FRFs of H deteriorated, the identification results became worse. As anticipated in Eq. (5), a small amount of noise in H can be amplified during the inverse operation of the matrix for the internal force computations, even though a singular value decomposition technique is used to solve the linear algebraic equations. Furthermore, from Tables 1-4, the noise included in the FRFs between external force and connection points had more negative effects on the identification results than the noise contained in the FRFs between connection and response points. In summary, one can conclude that when there was random noise in the FRFs, the

Table 1. Identification results with contaminated H matrix (Case I).

Noise Level (%)		0%	10%	20%
Stiffness (×$E8$ N/m)	K1	1.000	0.992	1.005
	K2	1.000	0.989	1.012
	K3	1.000	1.004	0.989
	K4	1.000	1.000	1.013
Damping Coefficient (×$E3$ N.sec/m)	C1	1.000	1.785	1.351
	C2	1.000	1.846	2.758
	C3	1.000	0.750	1.311
	C4	1.000	1.081	1.743

Table 2. Identification results with contaminated FRFs between external force point and connection points (Case II).

Noise Level (%)		0%	10%	20%
Stiffness (×$E8$ N/m)	K1	1.000	0.996	0.994
	K2	1.000	1.001	0.997
	K3	1.000	1.005	1.005
	K4	1.000	0.999	0.995
Damping Coefficient (×$E3$ N.sec/m)	C1	1.000	1.224	1.625
	C2	1.000	1.079	1.190
	C3	1.000	0.818	0.673
	C4	1.000	0.950	0.871

Table 3. Identification results with contaminated FRFs between connection points and response point (Case III).

Noise Level (%)		0%	10%	20%
Stiffness (×$E8$ N/m)	K1	1.000	0.993	1.006
	K2	1.000	0.991	1.004
	K3	1.000	1.002	0.999
	K4	1.000	1.000	1.000
Damping Coefficient (×$E3$ N.sec/m)	C1	1.000	1.314	1.149
	C2	1.000	1.155	1.234
	C3	1.000	0.803	1.041
	C4	1.000	0.962	0.961

Table 4. Identification results with all contaminated FRFs (Case IV).

Noise Level (%)		0%	10%	20%
Stiffness (×$E8$ N/m)	K1	1.000	0.978	1.012
	K2	1.000	0.995	0.998
	K3	1.000	1.027	1.005
	K4	1.000	0.993	1.009
Damping Coefficient (×$E3$ N.sec/m)	C1	1.000	2.481	2.573
	C2	1.000	2.122	2.783
	C3	1.000	0.126	0.975
	C4	1.000	0.948	1.224

identification procedure for the stiffness values was robust while the identification procedure for the damping coefficients was relatively weak.

3.2 Effects of the stiffness magnitude

If the magnitude of the joint stiffness is very large, the joint becomes an almost rigid connection. In this case, the influence of the joint parameters on the response may be indistinguishable because the compliance terms of Eq. (4) approach zero. These are added to the diagonal elements of the system matrix, which must be inversed to calculate the responses and sensitivity information. In this section, the effects of the stiffness magnitude on the identification results are investigated through numerical experiments.

First, the reference responses of the ladder-like structure described in the previous section were calculated by using the FE model of the overall structure. The joint stiffness values were increased from 1.0×10^8 to 1.0×10^{11} N/m, while the damping coefficients were fixed at 1.0×10^3 N·s/m to estimate the response variation with respect to the joint stiffness. Fig. 5 shows the calculated reference responses. Changes in the joint stiffness above a certain value, here approximately 1.0×10^{10} N/m, did not affect the reference response. Therefore, we can anticipate that beyond a certain threshold the identification problem does not have a unique solution because of the insensitivity of the reference response with respect to the joint parameters. Thus, the proposed procedure will yield an arbitrary solution among many possible solutions. To verify this point, the identification procedure was applied to one response shown in Fig. 5, which was selected as a reference response. The procedure was then repeated for the other responses. The identification results are plotted in Fig. 6. As expected, the identification results became worse for joint stiffness values above the threshold of 1.0×10^{10} N/m because the reference response was numerically the same for the entire region. Therefore, a prerequisite of the proposed method is that the reference response must be sufficiently sensitive to the joint parameters. However, if the only concern is to reproduce the reference response, it does not matter which joint parameter is used in this situation. In order to overcome the saturation problem successfully, one can expand the bandwidth of the index function and use stiffer substructures so that the index function becomes sufficiently sensitive with respect to the joint parameters.

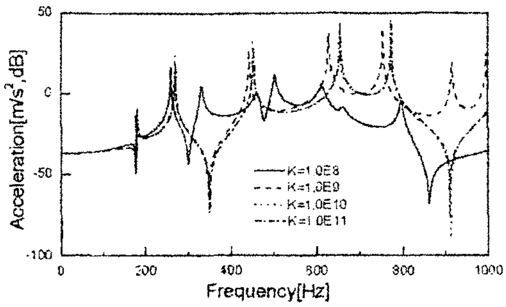
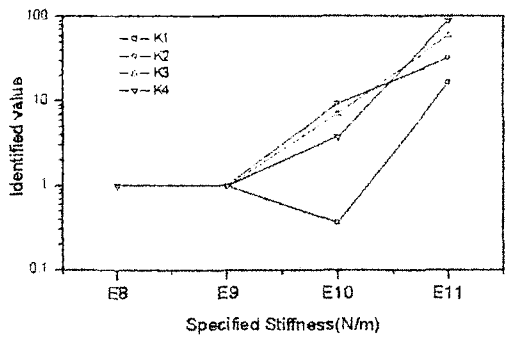
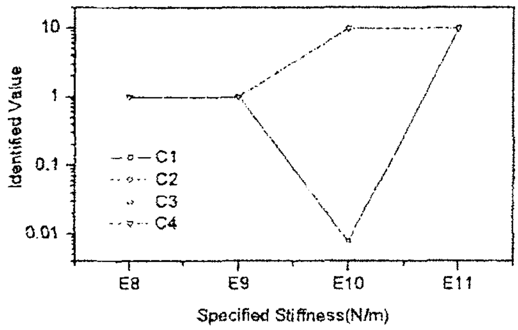


Fig. 5. The reference responses with respect to stiffness variation.



(a) Stiffnesses

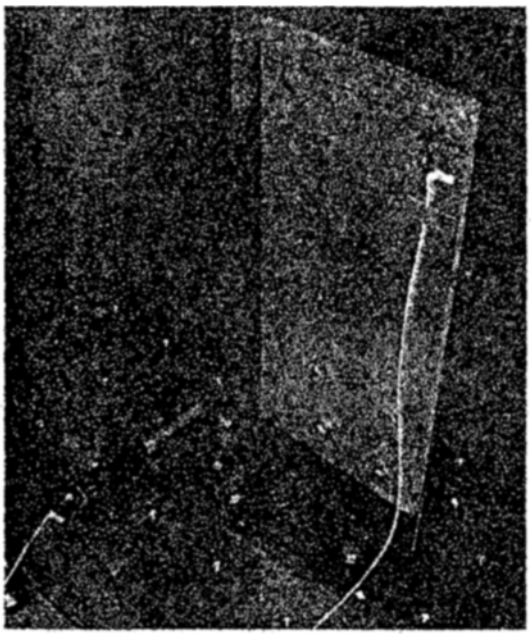


(b) Damping coefficients

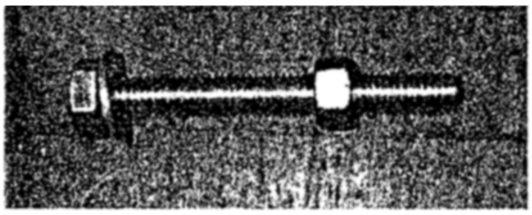
Fig. 6. Identified Stiffness values with different stiffness orders.

4. Application to a real structure

To verify the proposed identification method for joint structural parameters experimentally, we introduced the bolted structure shown in Fig. 7. The structure was composed of a steel jig fixed on a test bed and an aluminum plate. Four steel bolts and nuts connected the jig and the plate. The diameter and nominal length of the bolts were 8 and 37 mm, respectively. The object of this example was to identify the structural parameters of the bolts.



(a) Test set-up



(b) A connecting bolt and nut

Fig. 7. A bolted plate problem.

To construct the FBS model, the structure was divided into two substructures. The first substructure consisted of the test bed and the jig, and the second substructure consisted of the aluminum plate. The jig substructure was fixed on the test bed and had an excitation point and four irregularly distributed 10-mm diameter holes. The 205 × 450 × 10-mm plate substructure also had four 10-mm diameter holes and a reference response point, as shown in Fig. 7. The reference response and the FRFs of each substructure were measured for the FBS model from impact hammer tests using a small accelerometer. Scadas III front-end and Cada-X [24] software were used to acquire and transform the signal. Only one direction perpendicular to the plate was excited and measured in the experiment. The frequency band was 1000 Hz with 1-Hz increments. Fig. 8 shows a measured FRF for the plate substructure. With the measured FRFs,

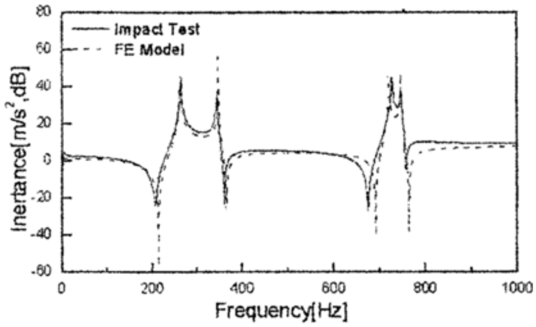


Fig. 8. Experimental and calculated FRFs of the aluminum plate.

an FBS model of the real structure is developed. In the FBS model, the transmissions of moments through the joints are assumed to be negligible, so that rotational FRFs are not used in the FBS model.

With the proposed method, the structural parameters of the bolts were identified by using the measured FRFs of each substructure. The identification index of Eq. (14) was used. The lower and upper limit frequencies were 2 to 1000 Hz, respectively. Here, we assumed that the joint parameters of each bolt were the same, as the bolts had the same appearance and specifications. Adding equality constraints to the optimization formulation enforced this assumption. Inequality constraints were also imposed to limit the lower and upper bounds of the joint parameters. To solve the minimization problem, the **constr** MATLAB function [23], which uses a quasi-Newton method, was employed. For comparison of identified results, a one-degree-of-freedom system with a known mass was introduced. Table 5 shows the identification results and a reference value identified from a vibration test of the one-degree-of-freedom system. It should be noted that there are many uncertainties in the "exact" values of joint parameters obtained from experiments if the experiments are not performed *in situ*. For example, the one-degree-of-freedom vibration test cannot include the effects of the fastening torque, nuts and washers, *etc.* Fig. 9 compares the reference response with the regenerated response from the proposed method. The identification results for the stiffness only are listed in Table 5 because the damping coefficients were always at the lower bound of the optimization problem, which was nearly zero. The identified stiffness value of the bolts was only 36% of the reference value. In addition, as shown in Fig. 9, the identified responses looked similar overall, but the first peak was not reproduced and several

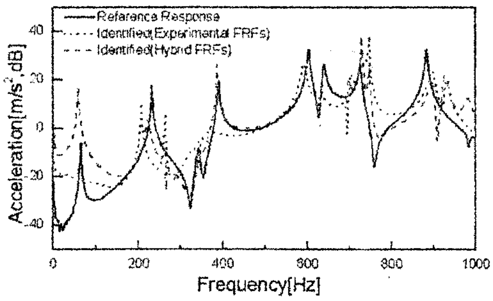


Fig. 9. Reference and identified responses of the bolted plate problem.

Table 5. Identification results of the bolted plate problem.

Method	Identified Stiffness (X(Product) E8 N/m)	Remarks
Experimental FBS Model	0.46	All FRFs from experiments
Hybrid FBS Model	1.31	FRFs of plate from an FE model
Reference Value	1.27	Identified from a one-degree-of-freedom vibration test

artificial peaks were generated around 250 and 700 Hz. Besides approximation errors in the FBS model such as the effects of ignored translational or rotational degrees-of-freedom, the disagreement in response may be owing to both biased and unbiased noises included in the FRFs. However, as numerically investigated in the previous section, unbiased random noise does not cause large error in stiffness identification results. Therefore, biased noises in experiments may lead to the deterioration of the identification results. For example, the connection points of substructures are not a point but a circle with finite dimension, so that experimental FRFs of substructure are acquired by averaging several FRFs which are obtained by impacting different locations around the circular connection point. These processes can cause inconsistency in FRF data set of a substructure.

An FE model of the aluminum plate was developed to examine the influence of the noise contained in the experimental FRFs on the identification accuracy. FRFs calculated from the FE model of the plate substructure were used to identify the joint parameters for the bolted plate problem. The FE model used four-node rectangular plate elements, as shown in Fig. 10. It was correlated to the experimental results in terms of the total mass, natural frequencies, and FRFs of the aluminum plate with free-free boundary conditions by updating the elasticity modulus of the aluminum. The

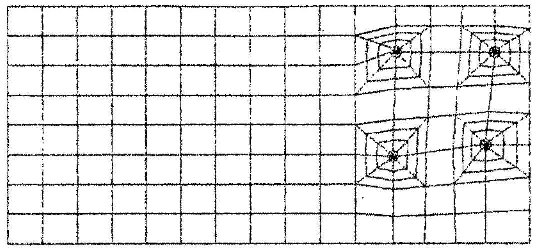


Fig. 10. An FE model for the aluminum plate.

Table 6. Comparison of eigenfrequencies for the aluminum plate with free-free boundary condition.

Mode No.	Eigenfrequency (Hz)		B/A (%)
	Test (A)	FE Model (B)	
1	263	260	98.9
2	342	349	102
3	728	728	100
4	746	761	102

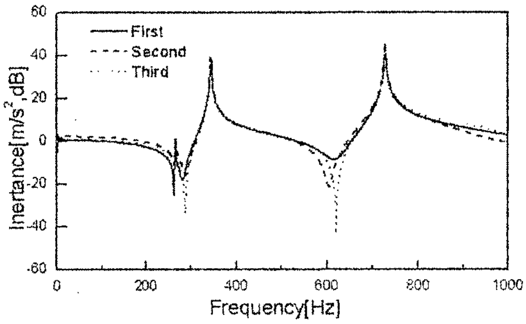


Fig. 11. Variations of a measured FRF in the aluminum plate substructure.

calculated and experimental FRFs are compared in Fig. 8, and the natural frequencies are listed in Table 6. The good agreement between the two models is readily visible. The joint parameters were identified again by using the calculated FRFs for the plate problem and the experimental FRFs for the other substructure. These are also listed in Table 5. Fig. 9 shows the reproduced response from using the identified results. The response from the hybrid FBS model represented the reference response better than the response from the experimental FBS model. In particular, the results of the hybrid model indicated a peak at around 70 Hz, although the level of the response was shifted, whereas the experimental model did not. This was an unexpected result because the difference between the FRF levels of the calculated and measured FRFs was very small around 70 Hz. It should be noted here that around peaks the system matrix H in Eq. (5) becomes a small value. In this situation, magnitudes of each FRF are very important both in absolute sense and in relative one. As one can assume that the calculated FRFs obtained from the FE model can keep consistency in a relative sense, the small amount of biased noise contained in the experimental FRFs could have been responsible for the large difference in the response of the FBS formulation through the inversion of the system matrix. Actually, the experimental FRFs show small variation which is not controllable. As an example, Fig. 11 shows the test variations of the measured FRFs for the aluminum plate. Each FRF in Fig. 11 was obtained from consecutive experiments and averaged five times by using the same test setup. Therefore, in viewpoint of data set consistency, FRFs of a substructure from a correlated FE model are more desirable than from experiments. Thus, hybrid approach in joint parameter identification would be a better solution in almost real structures.

5. Conclusions

Joint parameters, such as stiffness and damping coefficient values, can be identified through an optimization algorithm. Here, a joint parameter identification procedure using an FRF-based substructuring method and a gradient-based minimization algorithm was proposed. Uniformly distributed random noise was generated and added to the FRFs to test the robustness of the identification procedure. Furthermore, the effects of the relative magnitude of joint stiffness on the identification accuracy were explored over a wide range of values.

Numerical experiments showed that when the FRFs were contaminated with noise, the proposed procedure performed well when identifying the joint stiffness values, but it did not perform well when identifying the damping coefficients. Parameter studies of the relative magnitude of the joint stiffness values showed that the identification procedure yielded an arbitrary solution if the index function was not sufficiently sensitive to the joint stiffness values. This was because the reference response was saturated for very high joint stiffness values so that the inverse problem did not have a unique solution.

The proposed identification method was applied to a real problem, in which the structural parameters of bolts used to connect plates were identified. The identification results show that the proposed method can be applied successfully to real structures. In addition, it is desirable to use the FRFs calculated from an FE model of the substructures because of the consistency of the FRF set.

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